MATH 147 Review: The Modulus

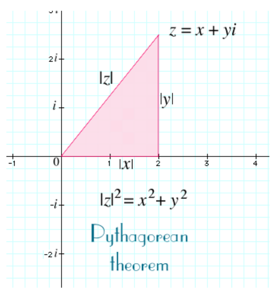
Facts to Know

The Modulus for Real Numbers

Let x be a real number.

Define $|x| := \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

The Modulus for Complex Numbers



Let $z = x + yi$

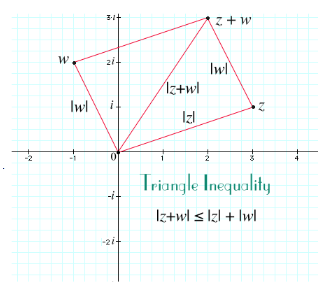
Define $|z| := \sqrt{x^2 + y^2}$

Basic Properties

Let $z = x + yi$, $w = a + bi$

- nonnegativity $|z| \geq 0$
- definiteness $|z| = 0$ if and only if $z = 0 + 0i$
- homogeneity $|z \cdot w| = |z| \cdot |w|$

Triangle Inequality



$|z+w| \leq |z| + |w|$

Reverse Triangle Inequality

$\big| |z| - |w| \big| \leq |z + w|$

for reals

for complex numbers

Examples

1. Compute $|z+w|$, $|z|$, and $|w|$ where $z = 1-3i$ and $w = 1+4i$. Verify that the triangle inequality and the reverse triangle inequality hold.

$$\big|\, |z| - |w| \,\big| \leq |z+w| \leq |z| + |w|$$

$$1)\ |z+w| = |(1-3i) + (1+4i)| = |2+1i| = \sqrt{2^2+1^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$|z| = |1-3i| = \sqrt{1^2+(-3)^2} = \sqrt{1+9} = \sqrt{10}$$

$$|w| = |1+4i| = \sqrt{1^2+4^2} = \sqrt{1+16} = \sqrt{17}$$

$$\left|\sqrt{10} - \sqrt{17}\right| \overset{?}{\leq} \sqrt{5} \overset{?}{\leq} \sqrt{10} + \sqrt{17}$$

2. Compute $|z \cdot w|$ where $z = 1-3i$ and $w = 1+4i$.

$$z \cdot w = (1-3i)(1+4i)$$

$$= (1+12) + (-3+4)i$$

$$= 13 + i$$

$$|z \cdot w| = |13+1i| = \sqrt{13^2+1^2} = \sqrt{169+1}$$

$$= \sqrt{170} = \sqrt{17 \cdot 5 \cdot 2}$$

$$= |z| \cdot |w| = \sqrt{10} \cdot \sqrt{17} = \sqrt{170}$$